1858MNRAS..19....4R

its variability. However, on July 3d of this year, while planet-seeking with the Smythian telescope, I was struck by finding it in the position assigned by Argelander, and a full 8th magnitude. Since that date the following changes have been recorded, not simply by estimation, but by the more certain and satisfactory plan now usually adopted of comparing with adjacent stars of assumed magnitudes:—

1858, July 3,	8·2 magnitude	1858, Aug. 8,	9·6 magnitude
11,	8·0 ,,	31,	11·2 ,,
19,	9·0 ,,	Sep. 5,	11·4 ,,
29,	9·3 ,,	Oct. 5,	12·8 ,,
Aug. 1,	9·4 ,,	Nov. 9,	invisible (under 12·5)

It appears most probable that if the variation be tolerably regular, seven maxima have occurred between the years 1849 and 1858; in which case the period will not differ much from 465 days, and the next maximum will fall due in September or October, 1859.

The position of R *Sagittarii*, as determined by micrometrical comparison with A.Z. 227·120, are found to be, when reduced to 1860, R.A. $19^{h}\ 8^{m}\ 28^{s}{\cdot}62$; South Dec. $19^{\circ}\ 32'\ 59''{\cdot}2$. That given by Argelander, reduced to the same epoch, being R.A. $19^{h}\ 8^{m}\ 28^{s}{\cdot}49$; and South Dec. $19^{\circ}\ 32'\ 57''{\cdot}4$.

2 South Parade, Oxford, Nov. 11, 1858.

On the Value of the neglected Terms in the Ordinary Expression for the "Equation of Equal Altitudes." By J. Riddle, Esq.

In order to elucidate what follows it will be necessary to refer for a moment to the expression erroneously used in the calculation of the correction, known under the designation of the "Equation of Equal Altitudes," viz.,

$$-e = c \cot p \cot h - c \cot l \operatorname{cosec} h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Where p = polar distance of the sun, l = colatitude of the place of observation, h = half the elapsed time between the A.M. and P.M. observations of equal altitudes of the sun, c the change of the sun's declination in the time h, and $-e$ the equation of equal altitudes.

This expression (1) is obtained directly by the differentiation of the fundamental one in Nautical Astronomy.

$$\left\{\begin{array}{c} \cos z = \cos p \cos l + \sin p \sin l \, . \cos h \\ z = \text{zenith distance: and } h = \text{hour-angle or meridian distance.} \end{array}\right\}$$

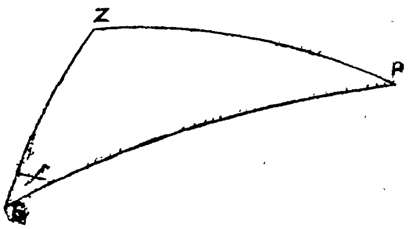
1858MNRAS..19....4R

Considering p and h only to change, it (1) is in fact only the value of $\frac{dh}{dp} \cdot c$, or the first term in the total difference which the change of the sun's declination produces in the hour-angle h. And this, in all our elementary treatises on Astronomy, is employed as a sufficient correction of the middle instant indicated by the chronometer, between the A.M. and P.M. observations of equal altitudes of the sun, to deduce the time shown by the chronometer at noon; and this, too, without any allusion to the value or *no* value of the neglected terms.

If, however, the next term be required,

$$\frac{d^2 h}{dp^2} \cdot \frac{c^2}{2},$$

it is by no means so easily obtained, in a practicable form, by a second differentiation of the already determined value of $\frac{dh}{dp}$; a convenient mode of attacking it has therefore some interest. There is not much difficulty in reducing the value of $\frac{dh}{dp}$ to the more concise form of $-\frac{\cot S}{\sin p}$; where S represents the angle at the sun between the zenith distance and polar distance; and looking at the geometry of the problem, this is the simplest form in which it presents itself, and therefore



$$-e = -\frac{\cot S}{\sin p} \cdot c = \frac{dh}{dp} \cdot c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)^*$$

Commencing then with this,

$$\frac{dh}{dp} = -\frac{\cot S}{\sin p}$$

and differentiating

$$\frac{d^2 h}{dp^2} = \frac{\operatorname{cosec}^2 S \,.\, \sin p \,.\, \frac{dS}{dp} + \cot S \,.\, \cos p}{\sin^2 p}$$

But since $\frac{dh}{dp} = -\frac{\cot s}{\sin p}$ $\therefore$ $\frac{ds}{dp} = -\frac{\cot h}{\sin p}$

(for S and h are the two angles of the spherical triangle adjacent to p),

$$\therefore \frac{d^2 h}{dp^2} = \frac{-\operatorname{cosec}^2 S \,.\, \cot h + \cot S \,.\, \cos p}{\sin^2 p}$$

$$= -\frac{(1 + \cot^2 S) \cot h - \cot S \cos p}{\sin^2 p}$$

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* See the note at the end of this paper.

1858MNRAS..19....4R

$$= -\frac{\cot h}{\sin^2 p} - \frac{\cot^2 S}{\sin^2 p}\cdot\cot h + \frac{\cot S}{\sin p}\cdot\cot p$$

And multiplying by $\frac{c^2}{2}$

$$\frac{d^2 h}{d p^2}\cdot\frac{c^2}{2} = -\frac{\cot h}{\sin^2 p}\cdot\frac{c^2}{2} - \frac{\cot^2 S}{\sin^2 p}\cdot\frac{c^2}{2}\cot h + \frac{\cot S}{\sin p}\,\frac{c^2}{2}\cdot\cot p$$

but $-\frac{\cot S}{\sin p}\cdot c = -e \;\therefore\; \frac{\cot^2 S}{\sin^2 p}\cdot\frac{c^2}{2} = \frac{e^2}{2}$

And therefore

$$\frac{d^2 h}{d p^2}\,\frac{c^2}{2} = -\frac{\cot h}{\sin p}\cdot\frac{c^2}{2} - \frac{e^2}{2}\cdot\cot h - \frac{e\,c}{2}\cot p$$

or,

$$= -\tfrac{1}{2}\cot h\left\{\frac{c^2}{\sin^2 p} + e^2\right\} - \frac{e\,c}{2}\cot p \;.\;.\;.\quad (3)$$

And this multiplied by sin 1″ will be the value of the second term of the difference when c and e are estimated in seconds.

In attempting to seek the largest values of the expressions (1) and (3), we are at once forced to consider the practical limits of the application of the problem. For at the poles cot l, and therefore e, are infinite; and this points, however vaguely, to a limit of latitude beyond which the problem is not fairly applicable. I am disposed to fix this limit at 70°, as being quite high enough, at any rate, for ordinary navigation, leaving the higher latitudes for a special polar legislation. (Even though I have been informed by an indefatigable observer that he has used this method several degrees farther north.) This limitation of latitude also includes a maximum limit of h.

A second and inferior limit of h is pointed out in the expression (1); for as cot h enters into it h ought not to be very small, 2^{h} or 30°, I think, would be an extreme limit here.

But a third, and for the present purpose, an infinitely better limiting condition, is furnished by equation (2) which is the second form of equation (1), viz.,

$$e = \frac{\cot S}{\sin p}\cdot c \;.\;.\;.\;.\;.\;.\;.\;.\quad (2)$$

For if the angle S, between the zenith distance and the polar distance be very small, it will give rise to an "ill-conditioned" triangle, which is at all times to be avoided. S should hardly be less than 10°. This condition has also the advantage of including those before mentioned.

We are now prepared to enter upon the discussion of the values of terms (2) and (3).

For the first, viz., $\frac{\cot s}{\sin p}\cdot c = e$, to find its largest value we must take the smallest values of S and p and the largest

1858MNRAS..19....4R

possible value of c. For these we have S = 10° and $p = 66\frac{1}{2}°$, and as far as c is concerned the following considerations. The greatest hourly change of the sun's declination is about 60″ in March and September; and taking half the elapsed time, or h, between the A.M. and P.M. observations as 6^h (a large value in actual practice), we have $c = 360'' = 24$ seconds of time. These values of S, p, and c give $e = 148{\cdot}4$ seconds of time, or about *two minutes and a half*, certainly an excessively large value for the equation of equal altitudes.

Next for the examination of the several parts of the expression,

$$\sin 1'' \frac{d^2 h}{d p^2} \cdot \frac{c^2}{2} \quad \left\{ -\tfrac{1}{2} \cot h \left(\frac{c^2}{\sin^2 p} + e^2 \right) - \frac{e\,c}{2} . \cot p \right\} \sin 1'' \quad . \quad (3)$$

The large values of e and c above found give

$$e\,c = 3561{\cdot}6 \text{ seconds}$$

and the greatest value of $\frac{1}{2}$ cot p . sin 1″ = ·000001, and therefore

$$\frac{e\,c}{2} . \cot p . \sin 1'' = {\cdot}0035616 \text{ seconds of time,}$$

and may be dismissed as of no appreciable value in practice.

There remains still

$$-\tfrac{1}{2} \cot h \left\{ \frac{c^2}{\sin^2 p} + e^2 \right\} \sin 1''$$

Or, since $\sin^2 p$ varies only between the values 1 and ·8,

$$-\tfrac{1}{2} \cot h \left\{ c^2 + e \right\} \sin 1'' \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Here again $(c^2 + e^2)$, with the large values found for c and e, amounts to 22598·6 seconds, and taking h at 15° (only one-half of the inferior limit which I am disposed generally to assign to it),

$$\tfrac{1}{2} \cot h . \sin 1'' = {\cdot}000009$$

And hence the final value of (4) is ·2035; or nearly $\frac{2}{10}$ seconds of time is found for an excessively large value of the second term of the difference, or secondary correction, and it may therefore also be safely neglected in nautical practice.

NOTE.

The following are the reductions referred to in this paper:—

$$\cos z = \cos p . \cos l + \sin p . \sin l \cos h$$

∵ differentiating,

$$0 = -\sin p . \cos l + \cos p . \sin l . \cos h - \sin p . \sin l . \sin h . \frac{d\,h}{d\,p}$$

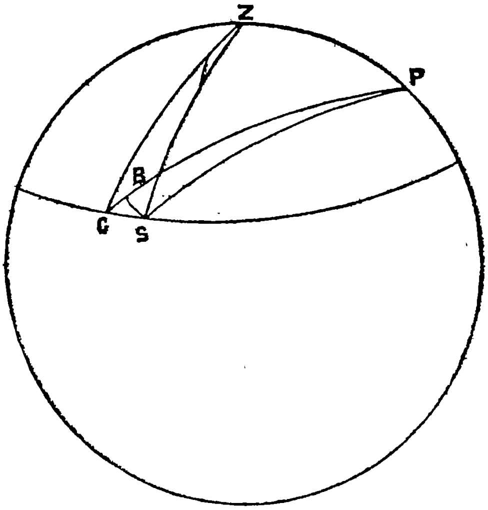
$$\because \frac{d\,h}{d\,p} = \cot p . \cot h . - \cot l . \operatorname{cosec} h$$

1858MNRAS..19....8.

$$= \frac{\cos p \,.\, \cos h}{\sin p \,.\, \sin h} - \frac{\cot l}{\sin h}$$

$$= \frac{\cos p \,.\, \cos h - \cot l \,.\, \sin p}{\sin p \,.\, \sin h}$$

$$= -\frac{\cot S \,.\, \sin h}{\sin p \,.\, \sin h} = -\frac{\cot S}{\sin p}$$



Or thus, let P C = the polar distance at the A.M. observation, and P A the polar distance at the P.M. observation, and A B a parallel of declination, as the polar distance is diminishing

$$C B = -2 C = 2 . dp,$$

And ∠ A P C is the increment of the hour-angle in consequence of this diminution in the polar distance = 2 . *d h*.

$$A B = A P B \sin A P = 2\, dh \sin p$$

$$\text{And } \frac{A B}{B C} = \cot \angle B A C = \cot \angle Z A P = \cot S$$

$$\therefore -\frac{dh}{dp} . \sin p = \cot S$$

$$\therefore \frac{dh}{dp} = -\frac{\cot S}{\sin p}$$

Greenwich Hospital, November 9th, 1858.

Results of the Observations of Small Planets, made with the Transit Circle, and of Donati's Comet, made with the Altazimuth, at the Royal Observatory, Greenwich, from July to October 1858.

(*Communicated by the Astronomer Royal.*)

Proserpine. ㉖

Mean Solar	Time of Observation.	Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1858, July 19	12 39 55·5	20 29 57·91	114 56 24·18
Aug. 7	11 7 39·1	20 12 21·00	115 53 17·49
9	10 58 6·8	20 10 40·20	115 57 5·38
11	10 48 42·8	20 9 7·78	116 0 19·12
16	10 25 17·4	20 5 21·33	116 6 19·76
Sep. 18	8 7 37·0	19 57 24·55	115 38 34·90